

MAS231

Assignment 6

Geometry II
Due on 10 March 2008

Write your name and student number at the top of your assignment before handing it in. Staple all pages together. Post the assignment in the Green Post-Box on the ground floor in the Maths building before 16:00 on Monday.

- Suppose that for some function $f(u, v)$ having partial derivatives f_u and f_v , the surface patch σ is given by $\sigma(u, v) = (u, v, f(u, v))$, $u \in \mathbb{R}$, $v \in \mathbb{R}$.
 - Show that σ is regular.
 - Find the unit normal $\mathbf{N}_\sigma$ and show that it points upwards.
 - Show that $\mathbf{N}_\sigma = (0, 0, 1)$ if and only if $f_u = f_v = 0$.
- Consider the surface patch $\sigma(u, v) = (u, v, \sqrt{u^2 + v^2})$, $U = \{(u, v) : u^2 + v^2 > 0\}$ for the cone $S = \{(x, y, z) : x^2 + y^2 = z^2\}$.
 - Does it cover the whole of S ?
 - Find the unit normal vector $\mathbf{N}_\sigma$ as a function of u and v . You may use your answer to question 1. Sketch the surface and indicate how $\mathbf{N}_\sigma$ points at a couple of typical points.
 - Show that $\tilde{\sigma}(r, \phi) = (r \cos \phi, r \sin \phi, r)$, $\tilde{U} = \{(r, \phi) : r > 0, 0 < \phi < 2\pi\}$ is another regular surface patch for S . Show that $\mathbf{N}_{\tilde{\sigma}}$ coincides with $\mathbf{N}_\sigma$ for the corresponding values of (u, v) .
- Consider the surface patch for the torus

$$\sigma(u, v) = ((a + b \cos(u)) \cos(v), (a + b \cos(u)) \sin(v), b \sin(u)),$$

$$U = \{(u, v) : 0 < u < 2\pi, 0 < v < 2\pi\},$$

- Show that
$$\mathbf{N}_\sigma = -(\cos(u) \cos(v), \cos(u) \sin(v), \sin(u)).$$
 - On the torus, sketch the curves (i) with $u = \pi$ and v varying, (ii) with $v = \pi$ and u varying and sketch how $\mathbf{N}_\sigma$ points at various parts of each curve.
- In the lectures we showed that $\mathbf{N}_\sigma = -\sigma$ for the standard surface patch for the sphere, i.e. the unit normal points inwards towards the centre of the sphere.
 - Another surface patch for the sphere is $\tilde{\sigma}(\tilde{u}, \tilde{v}) = (-\cos(\tilde{u}) \cos(\tilde{v}), -\sin(\tilde{u}), -\cos(\tilde{u}) \sin(\tilde{v}))$ with $\tilde{u} \in (-\frac{\pi}{2}, \frac{\pi}{2})$, $\tilde{v} \in (0, 2\pi)$. Show that $\mathbf{N}_{\tilde{\sigma}} = -\tilde{\sigma}$, i.e. the unit normal again points inwards towards the centre.
 - Using (a), explain why the sphere is orientable.